TEACHING TEAMS IN A MATHEMATICS COURSE: USING A PBL METHODOLOGY WHILE ACTION-RESEARCH ORIENTED

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Abstract

Given both the challenge posed by an EHEA/ECTS curriculum and the peculiar characteristics of the students’ type entering the university, a rethinking about the proper educational strategies requires to be implemented, along with the interactions that must be established to move towards a meaningful learning of the student, evaluating critical, reflective and interactive processes. In this way could be defined the mission of any teaching team. This paper reports a first-year mathematics experience based in problem-based learning to implement Linear Algebra projects. Workshops and cooperative groups are taken into account to develop the ongoing projects, which relate various subjects of the curriculum. Special emphasis is given to the facilitation tasks to be performed, while taking advantage of the strategy working group as to be mutual support among students in a group, or between different groups. In this way we see the need for such active methodologies that it is mandatory the involvement for more than a teacher (which means above all the fairness of the process from the teaching of the subject). So, teaching teams appear naturally in the learning-teaching process. Some at-present drawbacks are also outlined in our current syllabus.

Keywords: assessment rubric, constructivism, competencies, cooperative learning, ECTS, educational quality, EFQM model, EHEA, emotional intelligence, facilitation, project based learning, teaching teams, teamworking competency, teamwork training

Resumen

La misión de un equipo docente podría definirse como: Ante los nuevos Planes de Estudio que se avecinan, ante el reto que plantea el EEES/ECTS, y ante las peculiares características del tipo de alumnado que acceden a la Universidad se hace necesario un adecuado replanteamiento de las estrategias educativas que se aplicarán, junto con las interacciones que hay que establecer "de una manera obligada" para caminar hacia el aprendizaje significativo del estudiante, valorando los procesos críticos, reflexivos e interactivos. Esta comunicación presenta la experiencia llevada a cabo en primer curso de carrera en una asignatura de Álgebra Lineal, donde se utiliza la metodología de aprendizaje basada en proyectos para desarrollar, por medio de seminarios y grupos de trabajo colaborativo, un proyecto de curso, en el que se relacionan diversos asignaturas del currículum. Se hace especial énfasis en las tareas de acción tutorial que hay que desempeñar, al tiempo que se aprovecha la estrategia de trabajo en grupo para que haya un mutuo apoyo entre los estudiantes de un grupo, o bien entre grupos diferentes. De esta manera se ve la necesidad de trabajar este tipo de metodologías activas con la implicación, obligatoriamente, de más de un profesor (lo que implica ante todo la imparcialidad del proceso desde la parte docente de la asignatura).

Palabras clave: acción tutorial, aprendizaje basado en proyectos, aprendizaje cooperativo, calidad educacional, competencias, constructivismo, ECTS, EEES, equipos docentes, formación de equipos de trabajo, inteligencia emocional, matriz de valoración, modelo EFQM, trabajo en equipo
1. An introduction: the state of the art

The world today is characterized by rapid change. A number of factors such as globalization, the impact of information technology and communication, and the need for sponsoring and managing diversity necessitate an educational environment, significantly different [2]. The current trend (associated with the European Higher Education and Research Area; namely, EHEA) is that we are moving towards a learning society, and this involves the displacement of an education that focuses on education towards a learner-centred education [5]. Interest in the development of expertise in educational programs corresponds to an approach to education primarily focused on the student and their ability to learn, requiring to him more prominence and more compromise [6]. There is also a change in the concept of continuing education, where the individual needs to be able to handle knowledge, update, select what is appropriate for a given context, to understand what has been learned so that it can be adapted to new, and rapidly changing, situations [10, 43]. It will then need to reduce the use of class-present tasks and enhance the class-non-present tasks to 'teach to learn' so that the student can 'learning to learn' perceiving higher education as a further stage of 'learning over a lifetime' (long-life-learning) [17]. Clearly, the changing role of the teacher being the person who structured the learning process, the major player in education, as well as supervising the work of the students, whose knowledge he evaluated, in the vision centred in the student, the teacher is now a companion in the process of learning, which helps the student to achieve certain competences. While the role of the teacher remains critical, it moves increasingly toward an advisor, guide and motivator (i.e., this is the well-known metaphor of the thesis director and the novel researcher) [29, 33]. As a consequence, the ultimate objectives of the EHEA will involve major changes in the way of approaching the teaching strategy of the learning-teaching process (LTP). It is perceived that such changes significantly affect the manner in which the student’s curriculum in the daily life of a "subject". This poses a challenge for higher education institutions and universities in that it involves changing the educational system, revising the student’s and teacher’s time and working load,… Furthermore, the European Credit Transfer System (ECTS) will improve the quality of education and learning.

In addition, existing educational models show deficiencies of various kinds. The idea presented in this paper wants to be an approximation on how to address these challenges within a degree (namely, in a first course of Linear Algebra). In addition, the forthcoming emergence of new syllabus, considering a subject from an ECTS point of view (which means different teaching methodologies) [29], and working in a cooperative environment with other teachers –taking competencies (and/or professional attributions) as a reference, leads our students to be shown the benefits of this work philosophy. Also, it involves giving other information type –it is not excessive, where the efficient generation of meaningful knowledge is the cornerstone [39, 40]. Likewise, the change must be reflected in the assessment of the student, which should be focused on 'declarative knowledge' as a dominant reference, and often it only happens to include an assessment based on the skills, capabilities and processes closely related to work and to the activities that lead to the student’s progress (the need for continuous evaluation). Therefore, this paradigm shift should affect other aspects such as the approach of educational activities and teaching materials, which should lead to a variety of 'teaching situations' to encourage the student’s compromise. Also, this leads to the unavoidable conclusion that the LTP paradigm is changing.

While many learners of all ages lack necessary basic literacy competencies as well as higher order thinking competencies, today’s workplaces often demand high levels of both competencies sets. Economic, organizational, and technological forces have changed the nature of workplaces. Among these forces are globalization of the...
marketplace, democratization of workplace decision-making, synchronous production, new technologies, and multiple roles on most jobs [11, 12]. Consequently, the curriculum must deal with these challenges anywhere, anytime [3, 8]. In other words, Bloom-Anderson’s taxonomy model must be reconsidered to take all those new situations into account [7]. A lot experiences have been developed to include Project/Problem-Based-Learning (PBL) strategies since a very first-year course of the degree (elementary circuit analysis [9], automatic control courses [13], chemical engineering [25], computer network design [28], thermal and electrical engineering [30, 35], heat transfer [31], and so on). Because PBL methodologies, being quite active [18, 34, 20], can help to take advantage of all those challenges for future work environments [23, 24], it is highly recommended to include such contexts in our teaching syllabus, solving realistic problems [2, 5, 8, 10-12].

The resolution of projects/problems in Engineering and Experimental Sciences [20, 21], as a research-oriented teaching methodology, is useful in integrated cognitive-constructivist scenarios [4]. This technique is reported along the paper to show the student the major phases of a first-year Linear Algebra course applied to a typical situation of the degree, which has previously been characterized as a completely open problem [4, 18, 38]. Then, a problematic environment is focused to deal with PBL strategies in a mathematical situation to challenge the teamwork competency [44, 45]. While teachers, this methodology help us to deep in the everyday classroom action-research [39-41].

The paper structure is provided in the sequel. Section 2 describes what an open problem is from a mathematical viewpoint applied to a Linear Algebra first-year course. Section 3 considers PBL methodology applied to the LTP of the subject course, which is implemented in Section 4. Some results are discussed in Section 5; in particular, special attention is devoted to tutoring and facilitation (Section 6). Conclusions end the paper up.

<table>
<thead>
<tr>
<th>THEMATIC UNIT</th>
<th>LINEAR ALGEBRA (4.5 ECTS = 115 hours)</th>
<th>WEIGHT (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Finite dimension Vector Space Theory</td>
<td>10.00</td>
</tr>
<tr>
<td>2</td>
<td>Matrix Algebra</td>
<td>10.00</td>
</tr>
<tr>
<td>3</td>
<td>Systems of Linear Equations</td>
<td>15.00</td>
</tr>
<tr>
<td>4</td>
<td>Euclidean Vector Spaces</td>
<td>20.00</td>
</tr>
<tr>
<td>5</td>
<td>Spectral Theory</td>
<td>10.00</td>
</tr>
<tr>
<td>6</td>
<td>Practical Applications in the Mathematica® Programming Environment using algebraic and numerical techniques</td>
<td>15.00</td>
</tr>
<tr>
<td>7</td>
<td>Research Project (cooperative teams)</td>
<td>20.00</td>
</tr>
</tbody>
</table>

2. What does a problem mean?
Any situation can only be conceived as a problem to the extent that there is a recognition of it as such a problem, that is, it is unknown, and to the extent that, a priori, we do not have the solution: a situation for which there are no obvious solution. Accordingly, a problem can be defined as a situation such that there are no obvious solutions [18]. But it is desirable that students learn to solve problems raised at the university level, starting from open, and of interest, statements which include aspects from CTS (Science, Technology and Society) situations. The choice of problematic situations should be done in order to embody a challenge affordable by the students so that through interaction and the help of others, the student can participate in the learning process [2, 3, 8, 12, 14]. Solving a problem is to find a path where there previously was not a known one, finding a way to a difficult situation, to overcome an obstacle, to achieve a goal that could not be initially reached [17, 18]. This technique is...
proposed to be applied to address an open-ended problem, which will be the definition of a course project, each student has to raise in a course of Linear Algebra (Table 1) with competencies in an Industrial Electronics Technical Engineering degree, where the aim is to consider the student’s meaningful learning (Table 2).

### Table 2. The syllabus of Mathematical Grounds (competencies-based)

<table>
<thead>
<tr>
<th>COMPETENCY</th>
<th>MATHEMATICAL GROUNDS II (LINEAR ALGEBRA)</th>
<th>NUMBER OF CRITERIA FOR MEASURABLE OUTCOMES</th>
<th>WEIGHT (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Significant knowledge of the mathematical contents from an algebraic and numerical approach with practical applications from engineering fields</td>
<td>3</td>
<td>15.00</td>
</tr>
<tr>
<td>2</td>
<td>Numerical and symbolic modelling of linear continuous time-invariant dynamic systems via some professional programming environment</td>
<td>2</td>
<td>10.00</td>
</tr>
<tr>
<td>3</td>
<td>Obtaining of valid (reasoned and justified) conclusions from the results by efficiently managing the worked data</td>
<td>8</td>
<td>40.00</td>
</tr>
<tr>
<td>4</td>
<td>Design, implementation and management of a research project which will be produced in a collaborative team; then an scientific report will must be presented</td>
<td>5</td>
<td>30.00</td>
</tr>
<tr>
<td>5</td>
<td>To establish strategies and mechanisms to improve the learning quality through significant trainings and NICTs by considering policies such as sustainability, thrift, ethics, human rights, diversity, … and so on</td>
<td>3</td>
<td>10.00</td>
</tr>
</tbody>
</table>

The weight of the research work is important in monitoring the formative and summative assessment of the course: this is why special attention is devoted. To do so, students must apply the scientific method and the PBL methodology to develop the skills and attitudes (namely, competencies) that are inherent to the subject. It is not an exclusive method, but one more training methodology, while in the strategic resources within the daily teaching practice implementation. Project/problem solving includes a complex set of cognitive, behavioural and attitudinal components since problem solving is a multiple step process where the problem solver must find relationships between past experiences (schema) and the problem at hand and then act upon a solution (Figure 1).

The psycho-pedagogical model that is considered when implementing the subject syllabus is a Wittrock-Osborne’s in order the undergraduate to acquire any declarative (facts, concepts and principles) –know what, know that and know why, or procedural
knowledge (Figure 2) [7]. There are four stages in two hierarchical modes – informalization and structuring, to produce some conceptual change: firstly, the student must be aware of his/her misconceptions (elicitation phase) where ideas are tested and even, normally, decontextualized and destabilized; secondly, a new knowledge is achieved through information restructuring; third, as a consequence, the student actually takes and realizes this knowledge (invention phase) so as to be able to analyse it, to use it and to build new schemes and procedures (application phase).

![Figure 2. A reconsideration of the Wittrock-Osborne model for knowledge.](image)

However, in this mathematics syllabus there is a competency that must work at the same information search, time research, teamwork, cooperative teams and communication skills (Thematic Unit 7 in Table 1 and Competency 4 in Table 2) –see that there also is 5 assessment criteria to be considered as measurable outcomes. PBL methodology is quite a good strategy to apply this model into the syllabus but there some drawbacks (that will be revisited in Section 6) [14, 32]. The emphasis on project/problem solving is in addition to, and does not replace, the emphasis on basic literacy competencies in the schools. In national standards and tests, problem solving “raises the bar” from minimum competency to world-class competencies and professional attributions [19]. However, this is a first year mathematics course in an Engineering degree, so that problems must be well structured or at least moderately structured, because convergence to a right answer will help the students’ group through the diverse seminars. Next section is devoted to explain how PBL is applied in a Linear Algebra course (a concept map is given in Figure 3).

### 3. Project-Based-Solving Learning in a first-year Mathematics course

The learning methodology, which is suggested in the sequel, shows undergraduates develop the competences involved (Table 23), but a lot care must be considered (see Section 6), which is based on a cognitive-constructivist concept of the learning environment (see Figure 2) [38]:

A) The appropriate definition of the problem under consideration, with a discussion of the problem interest raised in connection with the descriptors of the course subjects;

B) The adequate knowledge of the physical mechanisms, describing the process being worked;
C) The statement of the objectives of the study (qualitative analysis stage), as well as the detection of the conditions, bounds and limitations that define the problem;
D) The generation of hypotheses for creative speculation;
E) The search for a solution, developing different methodologies and strategies;
F) The solution implementation, which will discover the group aptitudes and results, will be obtained;
G) The contrast interpretation of the results obtained;
H) Analyzing other possible extensions of the study; and
I) The development of the corresponding writing and speech, according to a particular pattern (for example, a scientific article), which will deploy the group and individual communication skills and attitudes [42].

In the specific case which is addressed in this paper (see Section 4), the above steps are applied to the photoelectric effect, analyzing the most salient features of this implementation (physical nature of the phenomenon under study, the objectives of the experience, the algebraic approach to the problem, the implementation of the resolution, and discussion of the proposed solution, along with the quantitative analysis of the results). As it deepens in the teaching methodology raised, the different results that are obtained are discussed, when analyzing the comments of the students that appear in the day-to-day register. Moreover, further examples of similar projects are reported (as work references), that are frequently used to analyze the way in which students acquire the course competencies, while pointing out the way in which the student's progress impacts on that achievement [14, 32].

(Figure 5. The photoelectric effect.

4. Application of PBL to the photoelectric effect
The photoelectric effect is a typical laboratory research that can be used to solve some interesting problems if it is tackled as an open problem (see Figure 5 and Table 3). The statement should follow this algorithm:

(S1) The Course Project is enunciated as an open problem by the team: The photoelectric effect sets up that the minimum voltage \( V \) to make an electron to leave a surface is a function of the frequency \( \nu \) (Hz) of the incidental radiation and of a certain function \( \phi \), which is characteristic of each surface. Looking at the existing literature, one must select several experimental data to estimate the values of the constants \( h \) (Planck's constant) and \( \phi \) so as to determine which such a surface is (Table 3).

<table>
<thead>
<tr>
<th>( \nu ) (Hz x10(^{-3}))</th>
<th>56</th>
<th>70</th>
<th>79</th>
<th>83</th>
<th>102</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_0 ) (V)</td>
<td>0.05</td>
<td>1.00</td>
<td>1.40</td>
<td>1.74</td>
<td>2.43</td>
<td>3.00</td>
</tr>
</tbody>
</table>

(S2) To take into account the PBL approach (Figures 2-4 and 6). A timetable must be provided to the student/group as a proposal of a working proposal (so, phases and steps must be give with detailed information on tasks, activities, outcomes, time estimates, facilitation and likely difficulties to appear, among others. The students’ team carries out the diverse stages of the research coached by the professor.)
(S3) The problem via a matrix, vector or numerical modelling to establish the dependence between the existing variables \( eV_0 = h\nu - \phi \) being \( \nu > v_1 = \frac{\phi}{h} \) the minimum frequency. At the same time, the work hypotheses are established (Figure 7). That is: students must look for the physical analysis of the problem, the data and result discrimination, the variables must be established, descriptors and keywords must be located to focus the solving, and an approach to the resolution method must be thought.

(S4) The theory key points must be revisited to have a good understanding: a concept map (Figure 7) is usually provided with every unit as an information help in the workshops/seminars, which are given every two weeks. In the meanwhile, each group can meet to cooperatively develop its work, or have personal work, according to the timing anticipated [15-16]. Sometimes, questions and drawbacks arise: students must look for tutoring –objectives, aims, difficulties are stated and activities suggested to overcoming them.

(S5) How the student can be coached to reach a true mathematical comprehension in order to determine the concept structure and the relationships of the involved mathematical units? (Figure 3).

(S6) Which are the student's main difficulties? The PBL approach is an adequate tool to develop the problems, the algebraic descriptors usually pose: the vector space idea, different algebraic tools for numerical computation, situations where high dimensionality appears, systems with overdetermined data, the spectral theory to summarize systems up, the analysis of the physical systems response, …

(S7) To establish the connections with other units of the syllabus or with other subjects of the degree: a good tutoring (namely, facilitation) is basic to adequately coach the team. At this point, some theoretical information search can be performed to complete and to round the project. Special questions can be posed to help in this task (see Section 6).
The problem solving is made by using a computer program (Matlab, Mathematica, Maple, …) to emphasize the results discussion and look for the appropriate explanations. Here, several problems can be detected which are related to the team/student’s working way. Consequently, additional objectives must be defined and the corresponding activities implemented to solve these difficulties (Figure 8).

\[
A = \begin{pmatrix}
0.056 & 1 \\
0.070 & 1 \\
0.079 & 1 \\
0.083 & 1 \\
0.102 & 1 \\
0.120 & 1
\end{pmatrix} \quad b = \begin{pmatrix}
0.05 \\
1.00 \\
1.40 \\
1.74 \\
2.43 \\
3.00
\end{pmatrix}
\]

solution = \left( {A^T A} \right)^{-1} \left( {A^T b} \right)

solution = \text{proj}_{\mathcal{L}(\mathcal{A}_1, \mathcal{A}_2)} b

for an orthogonal basis

Some keywords

- How do you calculate the range of any matrix? (equation matrix)
- How is introduced the idea of measurement in a vector space? (orthogonality problems)
- How to write the inner product in matrix formulation? (matrix algebra)
- What is the geometric interpretation of the approximation problem? (objective functions)
- How to automate the least-squares algorithm using optimality? (methods of matrix factorization)
- What does mean the approximate solution of an inconsistent system of linear equations in the least squares sense? (optimization, derivation and integration)
- How to get wave functions (typical in electronics) as a linear combination of sine and cosine functions, what practical reasons support the use of such functions? (composition of functions (dis) continuous)

Some questions to think about

Figure 7. Some theory key points to take into account.

Diagrama de dispersión

Modelo de regresión lineal
(mínimos cuadrados)

Datos observados
Datos teóricos
IC95
ICL95
Lineal (Datos observados)

Slope = 45.031 \frac{V}{Hz}

Ordinate at the origin = -2.224 V

Figure 8. From initial data to results previous to discussion and result verification.
The team should look for other technical situations to compare with: electronic, mechanical, chemical, economic, biological, hydraulic, neurophysiological ... models. This information search link together several subject of the course curriculum.

The solution is reported as a scientific article and an oral presentation is compulsory. The report includes: (1) a graphical plot, (2) the data preparation, (3) the mathematical model of the information, (4) a solving proposal, (5) the algebraic and numerical methods that have been used, (6) the results preparation, (7) the results obtaining, (8) the results interpretation and discussions and (9) the consistency of the supplied data and the analysis of the errors achieved. Also, proofs of the group meetings are required as an evidence portfolio with the resource management, that each team member must provide at the end of the course. A help template is provided to do it.

The student’s opinion is essential: so that the team must also write a short essay over the experience and each student must fulfi l an opinion questionnaire on the LTP. The following questions are considered (Figure 9):

- Is the student satisfied?
- The student, gets a significant training in an efficient manner in order to acquire new competences, skills and/or procedural knowledge?
- Does he/she apply these new strategies en his/her curriculum in an appropriate way?
- The changes that succeed in the student’s learning/teaching process, do they improve the considered teaching system?

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which must be given (outer evaluation). Also, this assessment is usually formative, except the last month when summative evaluation takes place.

<table>
<thead>
<tr>
<th>CONTENIDO A EVALUAR</th>
<th>PORCENTAJE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentación del informe</td>
<td>20</td>
</tr>
<tr>
<td>A. 1 El trabajo está relacionado con la actividad prevista</td>
<td>2</td>
</tr>
<tr>
<td>A. 2 El título refleja el contenido</td>
<td>2</td>
</tr>
<tr>
<td>A. 3 Las palabras clave clasifican bien el trabajo</td>
<td>2</td>
</tr>
<tr>
<td>A. 4 El resumen es comprensible y refleja el contenido</td>
<td>4</td>
</tr>
<tr>
<td>A. 5 El trabajo está bien presentado</td>
<td>3</td>
</tr>
<tr>
<td>A. 6 El trabajo está bien estructurado y organizado</td>
<td>4</td>
</tr>
<tr>
<td>A. 7 Los recursos tecnológicos aplicados son los adecuados</td>
<td>3</td>
</tr>
<tr>
<td>Contenido y detalle de la aplicación</td>
<td>50</td>
</tr>
<tr>
<td>B. 8 Los contenidos están bien expuestos y articulados</td>
<td>5</td>
</tr>
<tr>
<td>B. 9 Los métodos están claramente descritos</td>
<td>10</td>
</tr>
<tr>
<td>B. 10 Existe una relación con una clara razonamiento cognitivo progresivo</td>
<td>7</td>
</tr>
<tr>
<td>B. 11 Existe una clara concreción de los resultados</td>
<td>5</td>
</tr>
<tr>
<td>B. 12 Las conclusiones son claras y bien fundamentadas en los datos</td>
<td>10</td>
</tr>
<tr>
<td>B. 13 Las referencias mencionadas son las apropiadas</td>
<td>5</td>
</tr>
<tr>
<td>B. 14 Se usan los apéndices/anexos de forma adecuada</td>
<td>8</td>
</tr>
<tr>
<td>Originalidad y fundamentos</td>
<td>30</td>
</tr>
<tr>
<td>C. 15 Se usan con soltura los conceptos y contenidos de la asignatura</td>
<td>7</td>
</tr>
<tr>
<td>C. 16 El lenguaje y la sintaxis utilizados son los adecuados</td>
<td>7</td>
</tr>
<tr>
<td>C. 17 Se hace uso de una adecuada gestión de los recursos disponibles</td>
<td>10</td>
</tr>
<tr>
<td>C. 18 El contenido es original</td>
<td>2</td>
</tr>
<tr>
<td>C. 19 El trabajo es valioso</td>
<td>2</td>
</tr>
<tr>
<td>C. 20 Se hacen contribuciones personales interesantes</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Planified activities/tasks</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Lectures</td>
<td>8.11</td>
</tr>
<tr>
<td>2. Solving exercices in cooperative groups in the classroom</td>
<td>16.22</td>
</tr>
<tr>
<td>3. Problem workshop to solve in cooperative groups</td>
<td>6.31</td>
</tr>
<tr>
<td>4. Laboratory simulations</td>
<td>6.01</td>
</tr>
<tr>
<td>5. Research Course Project</td>
<td>3.00</td>
</tr>
<tr>
<td>6. Written report and oral presentation</td>
<td>4.51</td>
</tr>
<tr>
<td>7. Assessment</td>
<td>12.61</td>
</tr>
<tr>
<td>8. ICNTs use</td>
<td>10.21</td>
</tr>
<tr>
<td>9. Tutoring/Facilitation/Mentoring</td>
<td>36.04</td>
</tr>
</tbody>
</table>

Figure 10. The Project Course assessment criteria and task weight as a function of competencies.

There are many examples that can be used to develop the PBL approach to work in a Research Course Project at the university in a first year course. Some examples (that can be coordinated with other subjects) are: the vibration analysis, the phase plane study, the approximation theory in a lot of environments—for instance, when tendencies are important, non-linear systems linearization, the discussion of the algebraic properties of (continuous time and discrete time) linear systems, … Those examples allow to follow the student's progress (formative evaluation) and also to discuss his/her development (in the sense of evolution) (additive evaluation) from the measurable outcomes.

The photoelectric effect is the basis for the production of electricity by solar radiation and the use of solar energy. The photoelectric effect is also used for the production of cells used in flame detectors boilers of large thermal power plants, and in photosensitive diodes such as those used in photovoltaic cells and electroscopes or electrometers. At present, the photochromic materials are used, apart from the derivatives of copper (now reduced use), the silicon, producing greater electrical currents. The photoelectric effect is also manifested in bodies exposed to prolonged sunlight. For example, particles of dust from the lunar surface acquired a positive charge due to the impact of photons. The charged particles repel one another rising.
from the surface and forming a tenuous atmosphere. Space satellites will also acquire a positive electric charge on their surfaces and the negative light in the darkened, so it is necessary to take into account cumulative effects of loading in their design. All those fields are applications environments that students should declare, for instance, in their oral presentations and/or written reports. They can obtained interesting study cases to improve their Course Project.

5. Some results: discussion. Strengths and improvement areas

In the previous sections an open problem to work in teams via the PBL methodology to get a Research Course Project has been reported. Its stages, structure and main characteristics have been outlined by using the scientific method. Namely, a technological situation must be modelled to focus the descriptors, the resources and the algebraic methodologies of a Linear Algebra syllabus. Also, the relationships with other subjects must be reflected as competencies are evolving while using the PDCA (Deming’s wheel) work system. In the sequel, some results are given to show the advantages of the PBL methodology.

In the course 2004-2005 student’s mean tutoring assistance was 1.24 hours (individual tutoring time); in the course 2005-2006 was 2.38 hours; in the course 2006-2007, was 1.78 hours; in the course 2007-2008, 1.98 hours; the course 2008-2009 has carried 3.24 times per student. However, not being great enough, the Research Course Project (being a team task) implies cooperative work and students take an active part in the workshop sessions [26]. This task is mainly off-class. The estimated time attendance $t_p = 3.55 \text{ hours (30 \%)}$ and estimated time of non-attendance of the student's work $t_{np} = 8.75 \text{ hours (70 \%)}$ (which implies a total time $t_{RCP} = 12.30 \text{ hours}$ -for course 2008-2009). With regard to time spent in the course timing is 12%. The student has agreed with the scheduled time, but initial objectives are too demanding. However, when analyzing the end results, polls show a substantial agreement with those initial objectives scheduled [38] (Figure 11). The 96.34 % has said that the tutoring session is an important task but only the 63 % of the students has used it as a regular activity.

Evaluation results are quite interesting (Figure 12). A lot of students take the Research Course Project with quite good results, although teams are not big (3 or 4 people per group) and many students don’t take continuous evaluation (the causes are very diverse). There exist significant differences in 2004/05 and 2006/07 versus 2005/06 and 2007/08: the first ones correspond to morning courses (more academic) and the
second ones to evening courses (students usually are working and they require a more practical treatment).

<table>
<thead>
<tr>
<th>Qualification</th>
<th>2004/05</th>
<th>2005/06</th>
<th>2006/07</th>
<th>2007/08</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXCELLENT (Q &gt; 8.1)</td>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VERY GOOD (5.9 &lt; Q &lt; 8.1)</td>
<td>37</td>
<td>57</td>
<td>65</td>
<td>87</td>
</tr>
<tr>
<td>GOOD (3.2 &lt; Q &lt; 5.9)</td>
<td>26</td>
<td>43</td>
<td>23</td>
<td>13</td>
</tr>
<tr>
<td>BAD (Q &lt; 3.2)</td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Team number

Students' number

Registration number

Figure 12. Result summary of the Course Project additive evaluation.

The students' opinion increases every year. Results are quite good (Figure 13) and they invite to use the PBL approach. However, these results can be improved, and the students can help in such a task.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>POSITIVE ANSWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJECTIVES EXPLANATION</td>
<td>76%</td>
</tr>
<tr>
<td>FORMATIVE PROGRAM UNDERSTANDING</td>
<td>68%</td>
</tr>
<tr>
<td>COURSE PROJECT COMPETENCES</td>
<td>87%</td>
</tr>
<tr>
<td>SUPPLIED INFORMATION</td>
<td>78%</td>
</tr>
<tr>
<td>PROFESSOR'S HELP</td>
<td>89%</td>
</tr>
<tr>
<td>THE COURSE PROJECT IN THE SUBJECT</td>
<td>94%</td>
</tr>
<tr>
<td>HAS LEARNED TO WORK IN A TEAM?</td>
<td>75%</td>
</tr>
<tr>
<td>WORK ENVIRONMENT</td>
<td>96%</td>
</tr>
<tr>
<td>WOULD YOU WORK AGAIN WITH THIS PROFESSOR?</td>
<td>95%</td>
</tr>
<tr>
<td>GENERAL QUALIFICATION OF THE COURSE PROJECT</td>
<td>88%</td>
</tr>
<tr>
<td>PROFESSOR'S QUALIFICATION</td>
<td>87%</td>
</tr>
</tbody>
</table>

Figure 13. Results of the 2007/08 student's opinion questionnaire.

The way to solve the problem by the students will provide interesting information on how to use the capabilities of formal reasoning, and how to structure such relationships in his/her knowledge acquisition schema. For this reason, the problem must be solved manually and digitally; informatics tools should be used in the Mathematical Laboratory (in our case, the program Mathematica®). The analysis of this phase of the PBL timing has produced an affinity analysis of typical problems that often occur at this stage because (see Section 6). That is, an analysis based on the estimated point at which the student should be allows to obtain a better understanding of how to acquire the competences involved (Table 2) [1, 37].

Figure 14. The definition of the competency concept for implementation in any Teaching Planning.

6. A Challenge for improvement: the facilitation process
Taking into account the competency definition (Figure 14) one points out that active methodologies seem to be more appropriate to face the ECTS/EHSA binomial (i.e., academic competencies versus professional attributions) [3, 5, 44]. The student must do a lot of off-class work but, at the same time, he/she will need more help than ever. Consequently, a good facilitation support is to be ready [21, 37]. The implications on learning (student’s viewpoint) and teaching (teacher’s viewpoint) differ on the type of
the problem (Figure 15). That is, the PBL application reported in this paper must be an introduction to the PBL methodology because the subject is a first-year course and problems, though open, must be well structured. The duration is only four months and the undergraduate begins to walk in the competency acquisition process: the knowledge transfer will be near [18, 19]. However, he/she must also begin anytime.

The facilitation procedure (Figure 16) uses the errors learners make in problem solving as evidence of misconceptions, not just carelessness or random guessing (Figure 2). If possible, determine the probable misconception and correct it. The teacher must: ask questions and make suggestions about strategy to encourage learners to reflect on the problem solving strategies they use; do this either before or after the learner takes action; give practice of similar problem solving strategies across multiple contexts to encourage generalization [27, 36]; take into account the different learning styles while a well-balanced team [14-16] (Figure 17 shows a summary of some of the linkings between errors and misconceptions).

When developing a PBL Project course great care must to the facilitation task, above all in first courses (namely, this is the case where strengthening work in basic sciences is called for); namely, focusing on improving the student’s communication [42]. In this sense, facilitation must develop daily reflection [26]: pre-session (to present a focus concerning group dynamics so that facilitative questions should be used to start reflection), ordinary supervision session (with timeouts to discuss focus and to play diverse roles) and post-session (to facilitate reflections on the focus). Furthermore, facilitation implies tutoring and supervision (sometimes, even control—see Figures 4, 10, 16 and 17) to respond to student’s problems in terms of meta-competencies. Several dimensions are taken into account: the intellectual dimension, the personal dimension, the social dimension, the practical dimension (with several viewpoints: providing support, encouraging independence, developing the interpersonal) and
assessing research (formative assessment, creativity and originality, reliability and validity) [37].

However, the teacher’s role must also be considered from a leadership point of view: from hierarchy/autocratic/consultative to autonomy/functional/contractual via cooperation/negotiation/consultative. This implies that the student/teacher relationships ought to include six dimensions: the planning dimension (goal-oriented, aims, ends and means), the meaning dimension (cognitive understanding of experience), the confronting dimension (raising awareness to individual and group resistance), the feeling dimension (addressing emotional competence and incompetence), the structuring dimension (methodology of structuring experiences) and the valuing dimension (creating a support climate that celebrates individuals) [22, 37]. Figure 17 provides some of the tasks that are considered during the facilitation process of the Research Course Project.

![Figure 16. The complexity of facilitation [Kolmos et al., 2008:22].](image)

It can be seen the enormous complexity facilitation involves. If there is a few people only one teacher is enough, but course groups are normally bigger. The need to consider teaching teams is obvious to assure fairness in the facilitation process, above all if summative assessment is also involved. In other words, facilitation is a tool to challenge the student: for motivate, for involvement, for responsibility [40, 41, 45].

### 7. Conclusions

In the specific case which is addressed in this paper, the Problem Based-Learning (PBL) methodology, as research oriented, is applied to the photoelectric effect, analyzing the most salient features of this implementation (physical nature of the phenomenon under study, the objectives of the experience, the algebraic approach to
the problem, the implementation of the resolution, and discussion of the proposed solution, along with the quantitative analysis of the results). As it deepens in the teaching methodology raised, the different results that are obtained are discussed, when analyzing the comments of the students that appear in the day-to-day register. Moreover, further examples of projects (vibration analysis, …) are reported, that are frequently used to analyse the way in which students acquire the course competencies, while pointing out the way in which his/her progress impacts on that achievement.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Difficulties</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Prerequisites poorly assimilated</td>
<td>0º</td>
</tr>
<tr>
<td>1</td>
<td>Correctly apply the range concept</td>
<td>1º</td>
</tr>
<tr>
<td>1</td>
<td>Correctly apply the range concept</td>
<td>1º1</td>
</tr>
<tr>
<td>1</td>
<td>Correctly apply the range concept</td>
<td>1º2</td>
</tr>
<tr>
<td>2</td>
<td>Associate a vector space the measurement concept</td>
<td>2º</td>
</tr>
<tr>
<td>2</td>
<td>Associate a vector space the measurement concept</td>
<td>2º1</td>
</tr>
<tr>
<td>2</td>
<td>Associate a vector space the measurement concept</td>
<td>2º2</td>
</tr>
<tr>
<td>3</td>
<td>Describe an inner product by means of the associated Gram matrix</td>
<td>3º</td>
</tr>
<tr>
<td>3</td>
<td>Describe an inner product by means of the associated Gram matrix</td>
<td>3º1</td>
</tr>
<tr>
<td>3</td>
<td>Describe an inner product by means of the associated Gram matrix</td>
<td>3º2</td>
</tr>
<tr>
<td>4</td>
<td>Apply the Gram-Schmidt’s method</td>
<td>4º</td>
</tr>
<tr>
<td>4</td>
<td>Apply the Gram-Schmidt’s method</td>
<td>4º1</td>
</tr>
<tr>
<td>4</td>
<td>Apply the Gram-Schmidt’s method</td>
<td>4º2</td>
</tr>
<tr>
<td>4</td>
<td>Apply the Gram-Schmidt’s method</td>
<td>4º3</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º1</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º2</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º3</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º4</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º1</td>
</tr>
<tr>
<td>5</td>
<td>Organize the least squares method</td>
<td>5º2</td>
</tr>
</tbody>
</table>

| To think | How to deduce whether a given stock index has shown an upward or downward trend over a given time interval? |

Figure 17. Some questions that are posed to students along the help/tutoring sessions.

Problem resolution in Engineering and Experimental Sciences, as a research-oriented teaching methodology, is useful in cognitive-constructivist environments. This technique is reported in this paper to show to the student the major phases of an Algebra course applied to typical situations of the degree, which have previously been characterized as completely open problems. The world today is characterized by rapid change. A number of factors necessitate an educational environment, significantly different. The current trend is that we are moving towards a learning society, and this involves a displacement towards a learner-centred education. There is also a change
in the concept of continuing education, where the individual needs to be able to handle knowledge, update, select what is appropriate for a given context, to understand what has been learned so that it can be adapted to new, and rapidly changing, situations. Likewise, the change must be reflected in the assessment to the student, which should be focused on 'declarative knowledge’ and ‘procedural knowledge’ as dominant references. This paradigm shift should affect other features such as the approach of educational activities and teaching materials, which should lead to a variety of ‘teaching situations' to encourage the student’s compromise.

We analyse the possibilities of a course project based on the PBL methodology for teaching problem solving: its strengths and its weaknesses are emphasized. In other words, we conceive the possibility of proposing a vision for overcoming the teaching methods of problem solving in Science and Engineering, which in line with the model of learning as a research-oriented, leads to a methodological change in a way that the students arrive to be able to cope more successfully open-statement well-structured problems. The proposal is not characterized by dramatic changes in the structure of the education system but for *qualitative changes* in the teaching strategies used in the classroom, the activities proposed by the students and by the sequencing of conceptual and methodological contents.

The resolution of problems by students, accustomed to implement the proposed model, features closely the characteristics of the 'scientific work' and increases its effectiveness as resolvents. The procedural knowledge involves declarative knowledge specific to the area, and at the same time, the acquisition of the declarative knowledge is a process of construction that makes implicit or explicit use of procedural knowledge. It is therefore reasonable to expect that a further development of those procedures could lead to a more comprehensive learning and greater efficiency in the resolution. Since 2002 the authors have been working quite successfully with this student-centered method in our regular curricula and the enhancement is everyday in progress:

- Taking on personal responsibility for the learning process,
- Identifying the significance of previous knowledge,
- Cross-linking and integrating of learning and teaching contents,
- Dealing with literature an information,
- Experiencing, testing and internalising a method to solve problems,
- Learning and working in teams,
- Experiencing and understanding team processes,
- Reflecting own behaviour in a team,
- Leading discussions on a high level, and so on

At this point it is clear that the concept EHEA/ECTS implies a radical change of viewing the learning/teaching process. It means more work for teachers because it requires a careful and timely planning, with greater attention to detail (especially in the approach to assessment). This will require more dedication to the students (both individually and collectively). It also means more work for the student, which "must be a full-time worker" in the sense that they will have to organize their time and effort in a more autonomous way, than he/she does at present. Students should not begin relying on lectures, and take the teacher as a guide. Students will find a greater burden of documentation (not conceptual, but informative), allowing detailed criteria that are used throughout the methodological strategy of the course. This requires that the working groups are not too large (for their experience, the size should not exceed 40 persons per group), in order not to shoot time-exponentially teacher’s care and disposition, and all learning/teaching process suffers from the defects that have been carrying. The
active use of teaching teams should be an incentive to respond to this challenge and cause really meaningful learning of the students.

Problem solving owns three main characteristics: (1) problem solving is cognitive but is inferred from behaviour; (2) problem solving results in behaviour that leads to a solution; and (3) problem solving is a process that involves manipulation of or operations on previous knowledge. Some difficulties have appeared along the implementation because of being a first-year subject. To deal with this disadvantage a lot of facilitation must be conducted; otherwise, the student is not involved, not engaged. On the other hand, however, hard motivation and greater responsibility are attained if the student does accept the challenge, taking into account that cooperative teamwork (at this level a PDCA cycle is usually built) is being considered through the PBL development.

8. Acknowledgements
The authors are very grateful to the University of the Basque Country (UPV/EHU) for the partial support of this work through the Teaching Innovation Projects PIE UPV 2007/2009-27 –Estrategias constructivistas en un modelo de competencias para incidir sobre el aprendizaje significativo del alumnado mediante la readaptación de la metodología docente, and PIE UPV 2009/2010-1 –Las rúbricas de evaluación en formación por competencias, (ViceChair for Quality and Teaching Innovation through the Education Advisory Service –SAE/HeLaZ).

9. References.


